



# Rensselaer

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## Statistics Refresher & Intro to Labs

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**Data Analytics ITWS-4600/ITWS-6600/CSCI-4600 BCBP- 4600/ MGM-4600/MGMT-6600**

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Tetherless World Constellation  
Rensselaer Polytechnic Institute



# Contents

- Concepts and definitions
- Statistics Refresher
  - Population / Sample
  - Sampling Methods
  - Central Tendency
  - Frequencies / Probability Distributions
- R/RStudio installation



# Variable

- **Variable:** An attribute of a population or sample that can be measured.

e.g. weight, height, eye-color, pulse rate are some of the many variables that can be measured for people.



# Data

- Types of Data:
  - Qualitative (or Categorical)
  - Quantitative (numeric values)



# Data

- Qualitative:

Qualitative (or Categorical) data are descriptive, but not numeric.

Example: your eye-color, your gender, color of a vehicle, your birthplace



# Data

- Quantitative:

Quantitative data take on numeric values

- Discrete data take whole numbers (0,1,2,3..), representing countable things.

e.g. The number of times an employee is late to work, number of cars in the parking garage.

- Continuous data can take infinite possibilities within a range of numeric values (fractions, decimals are included..)

e.g. height of a person, weight of an apple, amount of time spent studying.



# Levels of Measurement

## Qualitative (or Categorical) data can be measured at the:

- Nominal Level: Values are just names, without any order (example: eye color)
- Ordinal Level: Values have some natural order, example: academic year (freshman, sophomore, ..), test grade, military rank

## Quantitative (data like numeric values) can be measured at the:

- Interval Level: Numeric data with no natural zero point; intervals (differences) are meaningful but ratios are not; can be negative, e.g. Temperature in Fahrenheit (or Celsius) degrees 80F is 20F hotter than 60F, but it is not 150% as hot.
- Ratio Level: Numeric data for which there is true zero; both intervals and ratios are meaningful; cannot be negative, e.g. weight, length, duration..



# Types of Data

| Type of Data        | Level of Measurement                                                                                                                    | Examples                                                |
|---------------------|-----------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------|
| <b>Categorical</b>  | <b>Nominal</b><br>(categories with no inherent order)                                                                                   | Eye color, ethnicity, academic major                    |
|                     | <b>Ordinal</b><br>(categories with inherent order)                                                                                      | Grade, academic year                                    |
|                     | <b>Binary</b><br>(2 categories; special case of above)                                                                                  | Electric charge                                         |
| <b>Quantitative</b> | <b>Discrete</b><br>(whole numbers)                                                                                                      | Size of household ( <b>ratio</b> )                      |
|                     | <b>Continuous</b><br>(can in theory take on any value in a range, although necessarily recorded to a predetermined degree of prevision. | Temperature C°/F° (no absolute zero - <b>interval</b> ) |
|                     |                                                                                                                                         | Age, height ( <b>ratio</b> )                            |



# Special values in data

- Fill value
  - Default
  - Missing value
  - Error value
  - Not-a-number (NAN)
  - Infinity
  - Null (NA)
- These vary by language/platform and usually require special attention (removal, debugging, etc.)



# Outlier

- An extreme, or atypical, data value in a sample.
- They should be considered carefully, before exclusion from analysis.
- For example, data values may be recorded erroneously, and hence they may need correction.
- However, in other cases they may just be surprisingly different, but not necessarily 'wrong'.



# Topics in statistics

- A statistic
- Statistics
- Population and Samples
- Sampling
- Distributions and parameters
- Central Tendencies
- Frequency
- Probability
- Significance tests
- Hypothesis (null and alternate)
- P-value
- Density and cumulative distributions



# Topics in statistics

- A statistic
- Statistics
- Population and Samples
- Sampling
- Distributions and parameters
- Central Tendencies
- Frequency

Today's class

- Frequency/Probability distributions
- Hypothesis (null and alternate)
- P-value
- Significance tests

Friday's class



# Statistic and Statistics

- **Statistic (not to be confused with Statistics)**

- Characteristic or measure obtained from a sample. (e.g. mean)

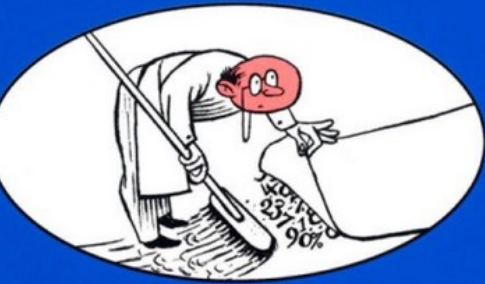
- **Statistics**

- Collection of methods for planning experiments, obtaining data, and then organizing, summarizing, presenting, analyzing, interpreting, and drawing conclusions.



# HOW TO LIE WITH STATISTICS

**Darrell Huff**  
**Illustrated by Irving Geis**



**Over Half a Million Copies Sold—  
An Honest-to-Goodness Bestseller**

## HOW TO LIE WITH STATISTICS (Huff, D. 1954)

There are three kinds of lies: lies, damned lies, and statistics.  
—Disraeli

Statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write.  
—H. G. Wells

It ain't so much the things we don't know that get us in trouble.  
It's the things we know that ain't so.  
—Artemus Ward

Round numbers are always false.  
—Samuel Johnson

I have a great subject [statistics] to write upon, but feel keenly my literary incapacity to make it easily intelligible without sacrificing accuracy and thoroughness.  
—Sir Francis Galton



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# What is "statistics"?

- The term "statistics" has two common meanings, which we want to clearly separate: **descriptive** and **inferential** statistics.
- But to understand the difference between descriptive and inferential statistics, we must first be clear on the difference between populations and samples.



# Populations and samples

- A **population** is a set of well-defined objects
  - We must be able to say, for every object, if it is in the population or not
  - We must be able, in principle, to find every individual of the population

e.g. All individuals of voting age in the US, all pixels in a multi-spectral satellite image
- A **sample** is a subset of a population
  - We must be able to say, for every object in the population, if it is in the sample or not
  - Sampling is the process of selecting a sample from a population

e.g. a number of voting-age individuals selected at random stratified by state, a set of pixels from known ground truth points

Courtesy Marshall Ma (and prior sources)



# Populations and samples

- **Population** : The complete set of actual or potential elements about which inferences are made
- **Sample** : A subset of the population selected using some sampling methods.

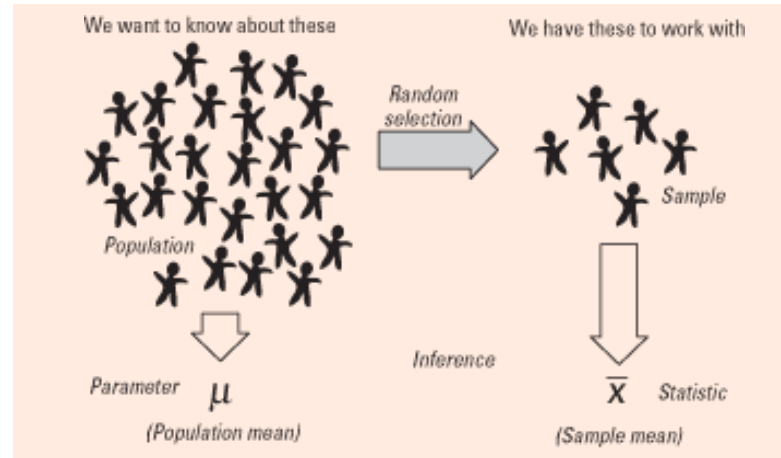


Image Credit/Reference: Quick Study Statistics

# Populations and samples

- A **population** = “all” of the data, if you can get it..
- A **sample** = “some” of the data, and you may not know how representative it is..

Courtesy Marshall Ma (and prior sources)



# Sampling Types

- **Random Sampling**

- Sampling in which the data are collected using chance methods or random numbers.

- **Systematic Sampling**

- Sampling in which data is obtained by selecting every  $k$ th object after a random starting point.

- **Convenience Sampling**

- Sampling in which data that is readily available is used.

- **Stratified Sampling**

- Sampling in which the population is divided into groups (called strata) according to some characteristic. Each of these strata is then sampled using one of the other sampling techniques.

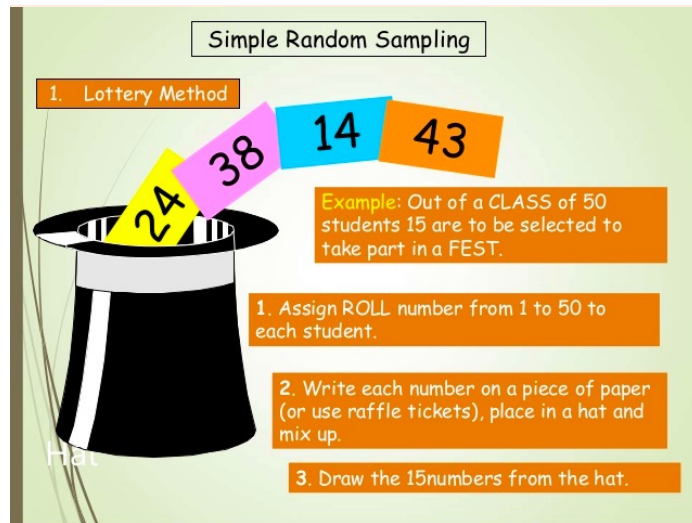
- **Cluster Sampling**

- Sampling in which the population is divided into groups (usually geographically). Some of these groups are randomly selected, and then all of the elements in those groups are included in the sample.



# Simple Random Sampling

A sample is selected so that each possible sample of the same size has an equal probability of being selected; used for most elementary inference

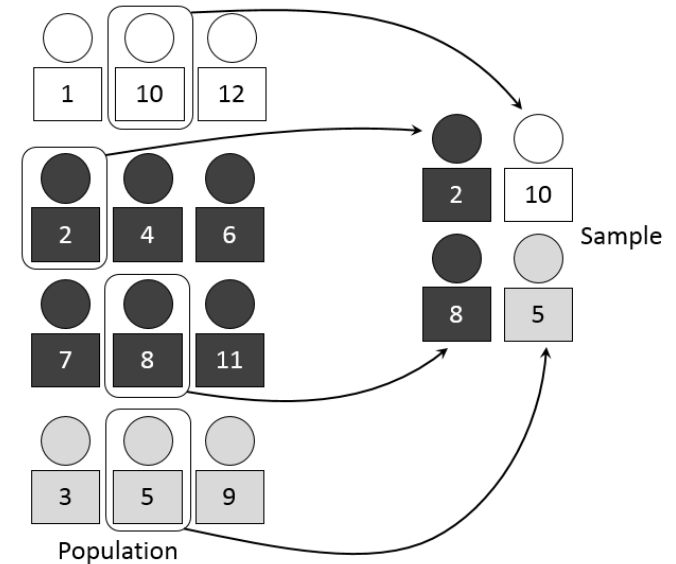


Courtesy: Quick Study Academic – Statistics [www.quickstudy.com](http://www.quickstudy.com)  
Reference: Quick Study Statistics  
Image Courtesy:  
<https://www.slideshare.net/mohammedzuhairy1/sampling-techniques-64917617>

# Stratified Sampling

The population is divided into strata based on some criterion, and a fixed number of elements from each stratum are selected for the sample (e.g. at random).

- Strata are internally homogenous.



[https://en.wikipedia.org/wiki/Stratified\\_sampling](https://en.wikipedia.org/wiki/Stratified_sampling)

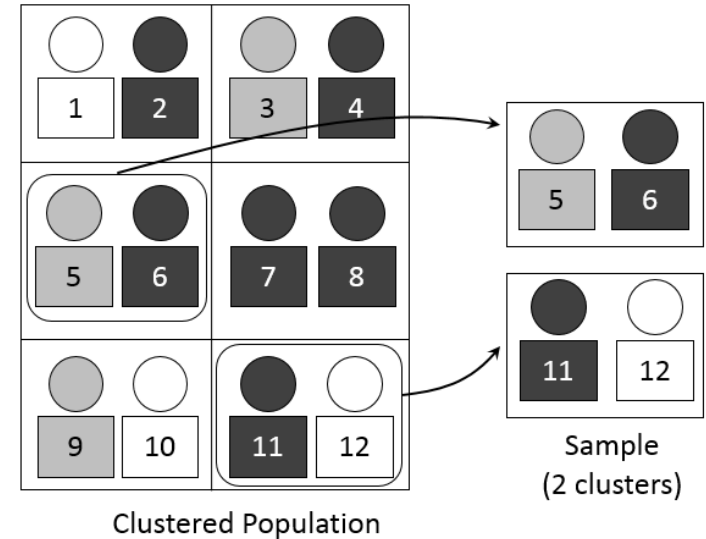
[https://en.wikipedia.org/wiki/Stratified\\_sampling#/media/File:Stratified\\_sampling.PNG](https://en.wikipedia.org/wiki/Stratified_sampling#/media/File:Stratified_sampling.PNG)

Image Credit: Dan Kernler – license: [CC BY-SA 4.0](https://creativecommons.org/licenses/by-sa/4.0/) - no changes

# Cluster Sampling

The population is divided into groups called clusters; some clusters are randomly selected, and every member in them observed.

- Clusters are internally heterogenous.

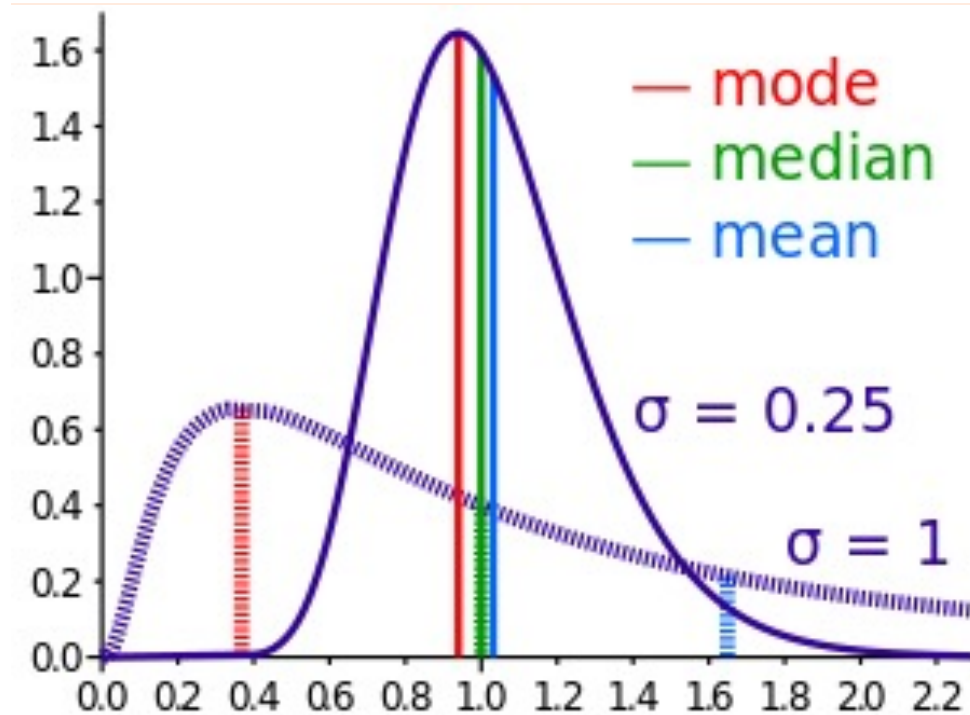


[https://en.wikipedia.org/wiki/Cluster\\_sampling](https://en.wikipedia.org/wiki/Cluster_sampling)

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# Central tendency – mean, median, mode



# Measure of Central Tendency

- **Mean:** The most commonly used measure of central tendency; commonly referred to as “Average”; sensitive to extreme values (sensitive to outliers)

- Population Mean

- Sample Mean

| Population Mean                         | Sample Mean                            |
|-----------------------------------------|----------------------------------------|
| $\mu = \frac{\sum_{i=1}^N x_i}{N}$      | $\bar{X} = \frac{\sum_{i=1}^n x_i}{n}$ |
| $N$ = number of items in the population | $n$ = number of items in the sample    |



# Measure of Central Tendency

- **Median:** Value that divides the set in 2 so the same number of observations lie on each side of it.
- ***The median is less sensitive to extreme values***
- For an even number, it is the average of middle two values.

1, 3, 3, **6**, 7, 8, 9

Median = **6**

1, 2, 3, **4**, **5**, 6, 8, 9

Median =  $(4 + 5) \div 2$   
= **4.5**

# Measure of Central Tendency

- **Mode:** Observation that occurs with the greatest frequency.



# Standard Deviation

## Population standard deviation of grades of eight students [\[ edit \]](#)

Suppose that the entire population of interest is eight students in a particular class. For a finite set of numbers, the population standard deviation is found by taking the **square root** of the **average** of the squared deviations of the values subtracted from their average value. The marks of a class of eight students (that is, a **statistical population**) are the following eight values:

2, 4, 4, 4, 5, 5, 7, 9.

These eight data points have the **mean** (average) of 5:

$$\mu = \frac{2 + 4 + 4 + 4 + 5 + 5 + 7 + 9}{8} = \frac{40}{8} = 5.$$

First, calculate the deviations of each data point from the mean, and **square** the result of each:

$$\begin{aligned}(2 - 5)^2 &= (-3)^2 = 9 & (5 - 5)^2 &= 0^2 = 0 \\(4 - 5)^2 &= (-1)^2 = 1 & (5 - 5)^2 &= 0^2 = 0 \\(4 - 5)^2 &= (-1)^2 = 1 & (7 - 5)^2 &= 2^2 = 4 \\(4 - 5)^2 &= (-1)^2 = 1 & (9 - 5)^2 &= 4^2 = 16.\end{aligned}$$

The **variance** is the mean of these values:

$$\sigma^2 = \frac{9 + 1 + 1 + 1 + 0 + 0 + 4 + 16}{8} = \frac{32}{8} = 4.$$

and the *population* standard deviation is equal to the square root of the variance:

$$\sigma = \sqrt{4} = 2.$$

[https://en.wikipedia.org/wiki/Standard\\_deviation](https://en.wikipedia.org/wiki/Standard_deviation)



# Ranges: z-score, Percentiles, Quartiles

- The **standard score** is obtained by subtracting the mean and dividing the difference by the standard deviation. The symbol is  $z$ , which is why it's also called a

z-score. 
$$z = \frac{x - \mu}{\sigma}$$

- **Percentiles (quantiles) (100 regions)**

- The  $k$ th percentile is the number which has  $k\%$  of the values below it. The data must be sorted.

- **Quartiles (4 regions)**

- Quartiles divide the data into 4 equal regions.
- Note: The 2nd quartile is the same as the median. The 1st quartile is the 25th percentile, the 3rd quartile is the 75th percentile.



# Five-number summary

The five-number summary is a set of descriptive statistics that provide information about a dataset. It consists of the five most important sample percentiles:

1. the sample minimum (smallest observation)
2. the lower quartile or first quartile
3. the median (the middle value)
4. the upper quartile or third quartile
5. the sample maximum (largest observation)

Read: [https://en.wikipedia.org/wiki/Five-number\\_summary](https://en.wikipedia.org/wiki/Five-number_summary)



# Frequencies

- The **Absolute frequency**  $n_i$  is the number of observations belonging to a category  $a_i$  or falling into a particular class  $c_i$ . The sum of all frequencies of all categories/classes is equal to  $N$ , the total number of observations:

$$\sum n_i = N$$

- **Relative frequencies**  $f_i$  are obtained by normalizing the individual frequencies to a total sum of 1.0 (or 100%, respectively). This way the frequencies become independent of the sample size and will be comparable to each other.
- Frequencies are usually delineated in a **frequency table** or displayed as a **histogram**.

e.g. Eye colors of 28 people

| Eye color | Abs. frequency | Rel. frequency |
|-----------|----------------|----------------|
| Brown     | 14             | 0.500 (50%)    |
| Gray      | 2              | 0.071 (7.1%)   |
| Blue      | 9              | 0.321 (32.1%)  |
| Green     | 3              | 0.107 (10.7%)  |



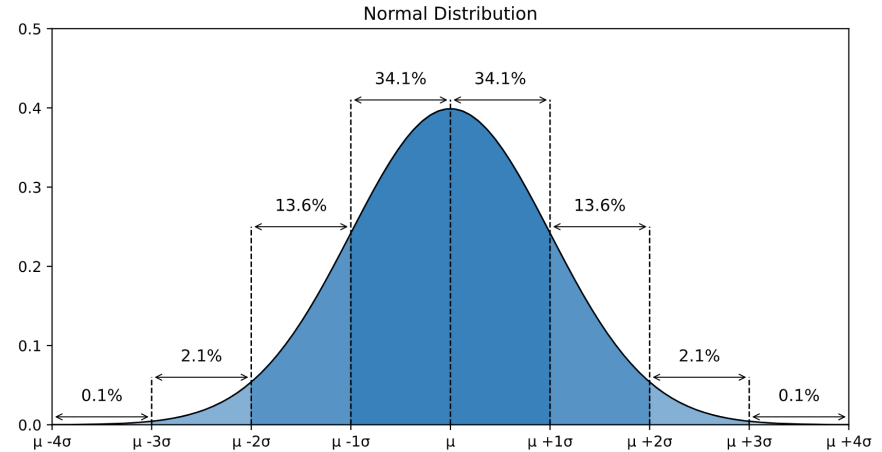
# Probability Distributions

- You should be familiar with classical probability distributions.
- Classical distributions have two properties:
  1. They describe shapes of frequency distributions that rise often in real world applications.
  2. They can often be described mathematically with few parameters.



# Normal Distribution

- The normal distribution implies tight bounds on the probability of lying far from the mean.
- 68% of the values must lie within one sigma (standard deviation) of the mean, and 95% within two times the sigma (standard deviation) and 99.7% lie within the three the sigma (standard deviation)



- Roughly 68.3% of the data is within 1 standard deviation of the average (from  $\mu - 1\sigma$  to  $\mu + 1\sigma$ )
- Roughly 95.5% of the data is within 2 standard deviations of the average (from  $\mu - 2\sigma$  to  $\mu + 2\sigma$ )
- Roughly 99.7% of the data is within 3 standard deviations of the average (from  $\mu - 3\sigma$  to  $\mu + 3\sigma$ )

Image Credit: W3C school:

[https://www.w3schools.com/statistics/statistics\\_normal\\_distribution.php](https://www.w3schools.com/statistics/statistics_normal_distribution.php)



# Parameter

- **Parameter:** A numeric measure that describe a population: parameters are usually not computed; but are inferred from sample statistics.
- Parameters are normally denoted using Greek symbols, whereas the corresponding statistics are denoted using Latin letters. Below you find a list of the most important parameters and the corresponding statistics.

| Parameter               |          | Statistic |
|-------------------------|----------|-----------|
| Mean                    | $\mu$    | $\bar{x}$ |
| Standard deviation      | $\sigma$ | $s$       |
| Correlation coefficient | $\rho$   | $r$       |



# Something to remember...

- Difference between “N” and “n”
- Population  $\longrightarrow$  N
- Sample  $\longrightarrow$  n



# Statistical Tests

- Let's say, the distribution of your observed (collected data) has a shape similar to that of a particular theoretical distribution (e.g. normal distribution), is that enough evidence that the points are normally distributed?
- There are “Statistical Tests” that can be used to rigorously test whether your obtained data reflect samples drawn from a particular distribution.
- We are going to talk about some of these statistical tests during the next couple of weeks...



# Getting started with analysis

- Examining the data visually
  - Which visualization techniques are suitable for one's data?
- Box plots
- Frequency plots
- Histograms
- Scatter plot diagrams
- ...



# Why visualization?

- Reducing amount of data
- Patterns
- Features
- Events
- Trends
- Irregularities
- Leading to presentation of data, i.e. information products



# Summary

- Types of data, levels of measurements
- Populations vs. samples, parameters vs. statistics
- Sampling types
- Measures of central tendency
- Absolutes / relative frequencies



# Next class: Friday 09/04

- Install R/Rstudio
- Getting started with programming in R



# Thanks!

