Rensselaer

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Distributional and Statistical Significance Tests

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Data Analytics

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Tetherless World Constellation
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Null and Alternate Hypotheses

- H0 – null
- H1 – alternate

- If a given claim contains equality, or a statement of no change from the given or accepted condition, then it is the null hypothesis, otherwise, if it represents change, it is the alternative hypothesis.

There is no significant difference between sales of 2 stores.

Two sets of values (variables) have identical distributions.

The values of a variable follow a normal distribution.

Accept or Reject?

- **Reject the null hypothesis if the p-value is less than the level of significance.**

- **You will fail to reject the null hypothesis if the p-value is greater than or equal to the level of significance.**

- **Typical significance 0.05 (!)**

Shapiro-Wilk

- “The Shapiro–Wilk test tests the null hypothesis that a sample x1, ..., xn came from a normally distributed population.”

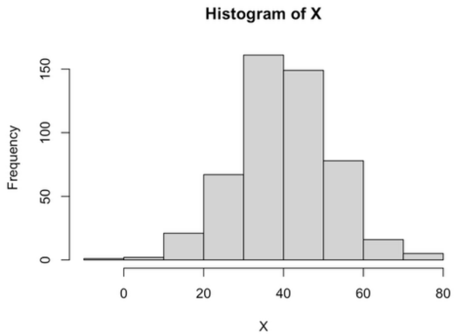
- H0: variable X is normally distributed

$$W = \frac{\left(\sum_{i=1}^{n} a_i x_{(i)}\right)^2}{\sum_{i=1}^{n} (x_i - \overline{x})^2},$$

shapiro.test(X)

Shapiro-Wilk normality test

data: X
W = 0.99638, p-value = 0.3175



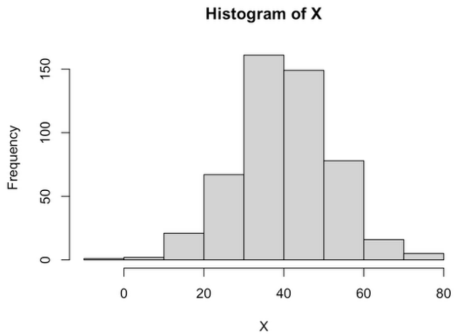
https://en.wikipedia.org/wiki/Shapiro%E2%80%93Wilk_test

Anderson-Darling (Normality Test)

- “The Anderson–Darling test is a statistical test of whether a given sample of data is drawn from a given probability distribution”

- H0: variable X is normally distributed

ad.test(X)



$$n \int_{-\infty}^{\infty} \left(F_n(x) - F(x)\right)^2 w(x)\, dF(x),$$

Anderson-Darling normality test

data: X
A = 0.26158, p-value = 0.7048

https://en.wikipedia.org/wiki/Anderson%E2%80%93Darling_test

Kolmogorov-Smirnov

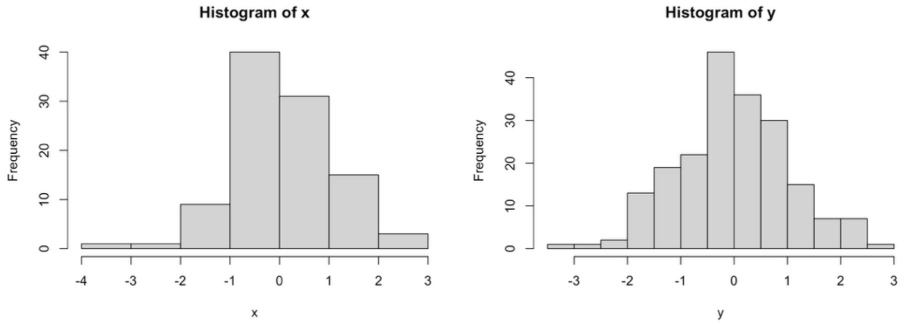
- “to test whether two samples came from the same distribution (two-sample K–S test)”
- H0: x and y are from the same distribution

ks.test(x,y)

$$D_n = \sup_x |F_n(x) - F(x)|$$

Asymptotic two-sample Kolmogorov-Smirnov test

data: x and y
D = 0.08, p-value = 0.787
alternative hypothesis: two-sided



https://en.wikipedia.org/wiki/Kolmogorov%E2%80%93Smirnov_test

Wilcoxon-Mann-Whitney

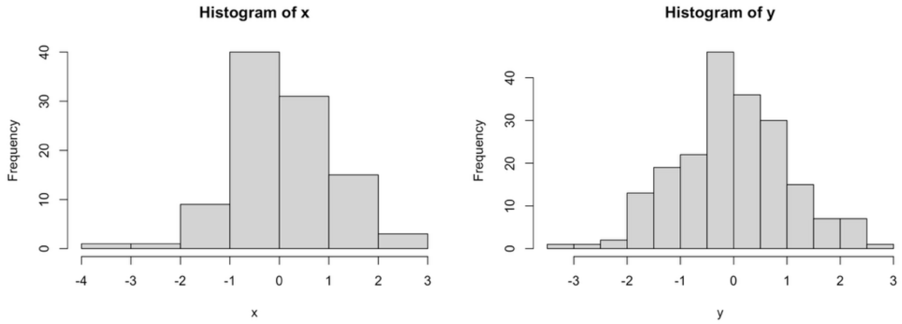
- “a nonparametric statistical test of the null hypothesis that randomly selected values X and Y from two populations have the same distribution”
- H0: x and y have the same distribution

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1, U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2$$

wilcox.test(x,y)

Wilcoxon rank sum test with continuity correction

data: x and y
W = 10220, p-value = 0.7566
alternative hypothesis: true location shift is not equal to 0



https://en.wikipedia.org/wiki/Kolmogorov%E2%80%93Smirnov_test

T-test

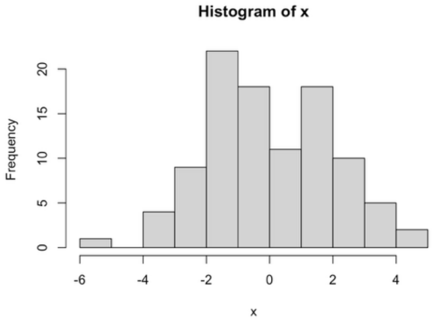
- “to test whether the difference between the response of two groups is statistically significant or not.”
- Assumes variables are normally distributed and have equal variance
- H0: x and y have the same distribution

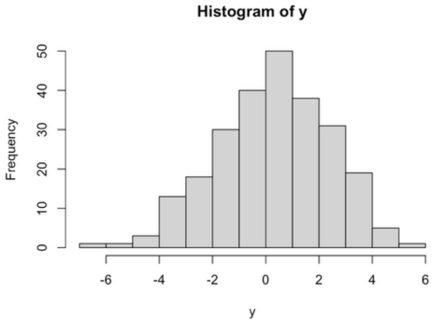
$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}},$$

t.test(x,y)

Welch Two Sample t-test

data: x and y
t = -1.2348, df = 194.41, p-value = 0.2184
alternative hypothesis: true difference in means is not equal to 0





https://en.wikipedia.org/wiki/Kolmogorov%E2%80%93Smirnov_test

F-test

- “It is used to determine if the variances of two samples, or if the ratios of variances among multiple samples, are significantly different.”

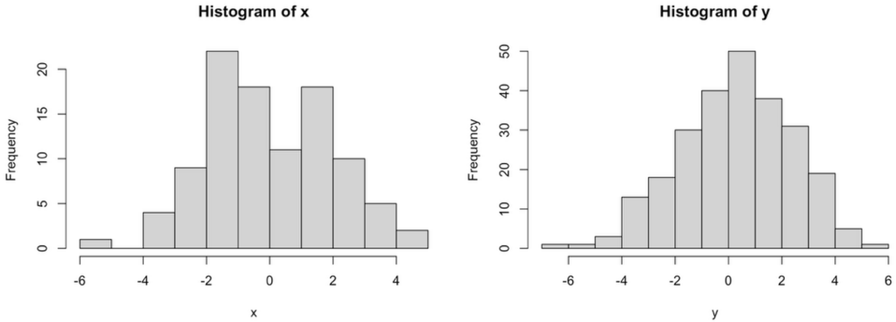
- H0: x and y have equal variance

$$F = \frac{S_A^2}{S_B^2}$$

var.test(x,y)

F test to compare two variances

data: x and y
F = 0.87183, num df = 99, denom df = 249, p-value = 0.4338
alternative hypothesis: true ratio of variances is not equal to 1



https://en.wikipedia.org/wiki/F-test

Thanks!

*** Have a good weekend!